Mapping with Kalman Filter on Riemannian Manifolds

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Abstract

The study of vibrating systems, particularly those equipped with geometrically consistent tuned mass dampers (GTMDs), plays a crucial role in understanding structural behavior and resilience against external forces. This research delves into the challenging domain of estimating parameters and states of vibrating structures adorned with GTMDs with precision. The application of dynamical systems in contemporary engineering necessitates a sophisticated modeling approach, especially when dealing with the complexities of structures on Riemannian manifolds. This paper introduces Kalman filters as a foundational tool for parameter and state estimation on Riemannian manifolds, particularly within the context of Lie Algebra. The research aims to bridge existing gaps in the literature related to parameter and state estimation in the context of vibrating structures with GTMDs.

1 Introduction

Structural dynamics, particularly the study of vibrating structures with geometrically consistent tuned mass dampers (GTMDs), is essential for earthquake engineering and structural design. The paper draws inspiration from the complexity of these systems and employs Kalman filters for parameter and state estimation on Riemannian manifolds. The use of dynamical systems in both natural and technological contexts is explored, emphasizing the need for a nuanced modeling approach. The introduction provides an overview of the research objectives and the significance of the study in advancing the understanding of structural dynamics.

1.1 Key Concepts and Literature Survey

In this section we are majorly focussing on introduction to the key concepts of structural vibrations, focusing on the application of Tuned Mass Dampers (TMDs) and Pendulum Tuned Mass Dampers (PTMDs). We explore the state of the art in modal parameter estimation and introduce the complexities associated with PTMDs.

The works of A. J. Roffel and Sriram Narasimhan [1] have addressed the need for comprehensive methodologies to estimate modal parameters while PTMDs are in service. In their paper, they present a method for time domain modal parametric identification of natural frequencies, mode shapes, and modal damping ratios of structures equipped with PTMDs. This research acknowledges the crucial role of PTMDs in structural dynamics, emphasizing their adaptive passive nature with mechanisms to adjust auxiliary frequency and damping. One key observation from the studies of Roffel and Narasimhan [1] is the inherent uncertainty associated with estimating the first modal damping. This uncertainty partly stems from the frequency-dependent behavior of the dampers. Interestingly, this uncertainty is notably reduced for the second mode, suggesting that the damper exhibits less frequency-dependent behavior at higher frequencies. This insight underscores the complexity of PTMDs and highlights the importance of a precise and robust estimation framework for modal parameters, especially in cases involving varying dynamic conditions.

Parallel to the advancements in structural dynamics, the field of differential geometry has gained attention in modeling non-linearities by confining parts of the model to Riemannian manifolds. The work of Søren Hauberg, Francois Lauze, and Kim Steenstrup Pedersen[2] introduces a novel algorithm that generalizes the unscented transform and the unscented Kalman filter for Riemannian manifolds. This pioneering research provides a generic optimization framework for these domains and demonstrates its robustness and convergence across various applications. In particular, the Riemannian unscented Kalman filter (UKF) is noted for producing smoother motion estimates, making it a promising tool for modeling complex structural vibrations with improved accuracy.

Building on this, the paper by Tripura, Panda, and Hazra [3], extends the horizon of real-time modal identification techniques. Their work introduces a novel approach that leverages first-order error-adapted eigenperturbation to enhance the accuracy and efficiency of real-time modal identification in vibrating structures. By incorporating differential geometry concepts from Pennec [3] into the identification process, this research represents an innovative step toward addressing the challenges posed by dynamic structural behavior.

1.2 Objective

The Study in this paper is carried out with the following objectives in mind

1. State and parameter estimation of systems evolving on Riemannian Manifolds, Eg: Chaotic Pendulum (3D Pendulum which has large swing angle) and Passive and Semi-Active TMD (Tuned Mass Dampers).

2. Optimisation and Parameter estimation of Pendulum Cart System evolving on SO(3) Manifold.

3. Mathematically model Geometrically consistent pendulum should capture all the displacements and rotations (including 3 translations, 3 rotations, and interaction coupled displacement).

2 Data and Methods

2.1 Dynamic Equation for Chaotic Pendulum on $SO(3)$ Manifold:

To investigate the dynamic behavior of a structure featuring a chaotic pendulum equipped with a geometrically consistent tuned mass damper (PTMD), we begin by defining the governing dynamic equation on the Special Orthogonal Group $SO(3)$ manifold:

$$I \cdot \dot{\omega} = -\omega \times I \cdot \omega + u \quad (1.1)$$

Where:

- I is the moment of inertia matrix.
- ω represents the angular velocity vector.
- $\dot{\omega}$ is the time derivative of the angular velocity.
- u represents the control input, incorporating the effects of the tuned mass damper and external forces.

2.2 Numerical Integration with Geometric Ito-Taylor 1.5 Method:

The dynamic equation is numerically solved over time using the Geometric Ito-Taylor 1.5 method[4]. This method proves advantageous for capturing the system's behavior accurately, considering both deterministic and stochastic components. The numerical integration process involves discretizing the time domain and updating the state variables (angular velocity and orientation) at each time step.

2.3 Incorporating Stochastic Elements:

Real-world scenarios often involve uncertainties and external disturbances. To emulate these conditions, stochastic elements are introduced into the simulations. Leveraging the capabilities of the Geometric Ito-Taylor 1.5 method, we model stochastic processes effectively. These stochastic components are included in the control input, simulating the impact of uncertain forces or disturbances on the system.

2.4 Parameterization of the PTMD:

To achieve geometric consistency with the tuned mass damper, the characteristics of the PTMD are parameterized within the control input u. This parameterization includes adjustments to the auxiliary frequency and damping properties of the PTMD in accordance with design requirements.

2.5 Simulation Output:

The output of the simulation yields time-series data representing the structural vibrations of the system. This encompasses the motion of the chaotic pendulum, the response of the PTMD, and any other pertinent parameters of interest. This carefully crafted data generation process facilitates the simulation of structures equipped with geometrically consistent tuned mass dampers under various conditions, enabling a thorough investigation into modal parameter estimation and structural response in a controlled and repeatable manner.

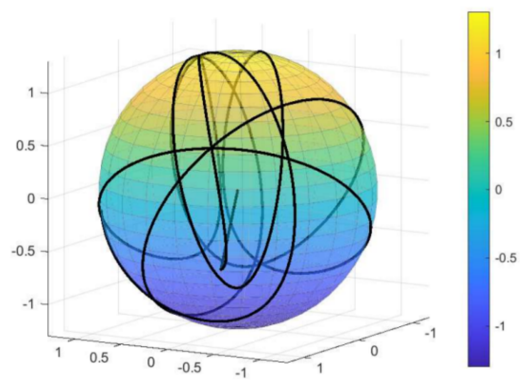


Figure 1: Trajectory of pendulum bob on SO(3) Manifold

2.6 Filter Design and Initialization

The state of the system is embedded in $SO(3) \times \mathbb{R}^3$ with left multiplication. The propagation noise covariance matrix, measurement noise covariance matrix, and initial uncertainty matrix are appropriately defined. The UKF is initialized with the chosen parameters and state[5].

2.7 Filtering

The UKF proceeds with a standard Kalman filter loop, involving state propagation and update steps based on received measurements. The estimates of the state and covariance are recorded along the trajectory.

2.8 Results

The results showcase the accuracy, robustness, and consistency of the UKF in estimating the position of the spherical pendulum, even in the presence of strong initial errors. Plots depict the position of the pendulum as a function of time, along with a 3σ interval confidence, demonstrating the convergence to the true state and the consistency of the filter.

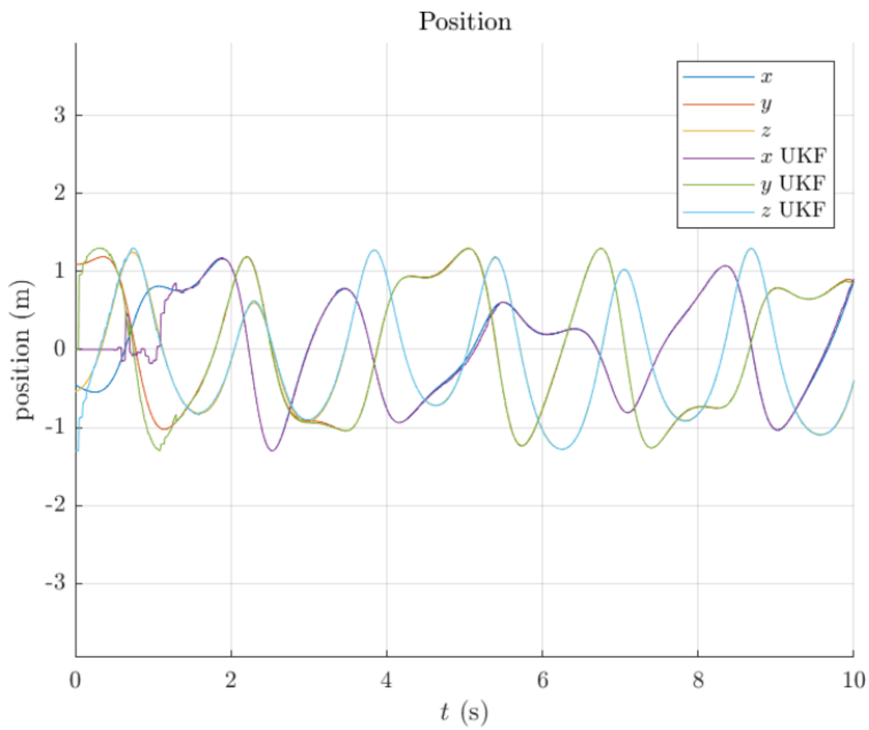


Figure 2: Position vs. Time for the spherical pendulum.

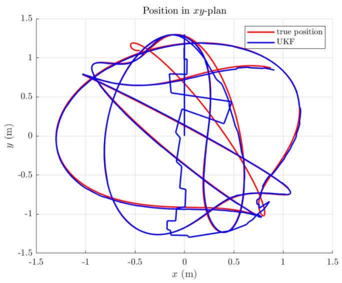
3 Applications and Future Directions

The successful application of the Unscented Kalman Filter (UKF) on parallelizable manifolds for estimating the position of a spherical pendulum opens up various potential applications and suggests avenues for further research.

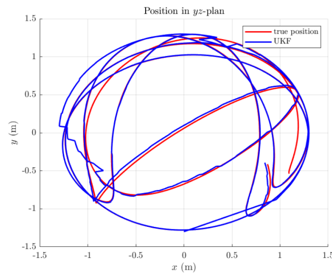
3.1 Applications

3.1.1 Structural Dynamics

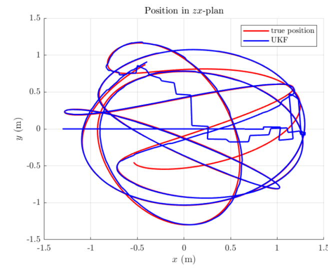
The application of the UKF on parallelizable manifolds has direct implications for structural dynamics. The accurate estimation of the position of a spherical pendulum showcases the potential of this methodology in monitoring and controlling the behavior of complex structures equipped with tuned mass dampers. This can have practical applications in the field of civil engineering, particularly in designing structures that can better withstand dynamic forces such as earthquakes and wind loads.



(a) Position of Pendulum in the XY plane.



(b) Position of Pendulum in the YZ plane.



(c) Position of Pendulum in the ZX plane.

Figure 3: Position of Pendulum in different planes.

3.1.2 Robotics and Autonomous Systems

The principles of state and parameter estimation on manifolds are crucial in the field of robotics and autonomous systems. The UKF's ability to handle the non-linear dynamics of a chaotic pendulum on the SO(3) manifold suggests its applicability in estimating the state of robotic systems operating in three-dimensional space. This includes scenarios where robots need to navigate complex environments and adapt to dynamic changes.

3.1.3 Real-time Systems

The feasibility of integrating the UKF into real-time systems is an area that warrants further exploration. Understanding the computational requirements, challenges, and benefits of implementing the UKF on parallelizable manifolds in real-time applications can have implications for a wide range of domains, from aerospace to medical devices.

3.2 Future Directions

3.2.1 Implementing the Kalman Filter on Tuned Mass System and in Complex Geometries $SO(3) \times R^2$

Extending the application of the Kalman Filter beyond the spherical pendulum, this subsection delves into the implementation of the filter on tuned mass systems. Tuned mass dampers play a vital role in mitigating structural vibrations, and employing the Kalman Filter can enhance the accuracy of estimating their parameters and states.

Furthermore, the exploration extends to complex geometries represented by the product manifold $SO(3) \times R^2$. This complexity arises in scenarios where structural components exhibit both rotational dynamics (governed by $SO(3)$) and translational dynamics (governed by R^2). Implementing the Kalman Filter in such intricate settings contributes to the understanding of multi-dimensional structural behavior and opens avenues for advanced control strategies.

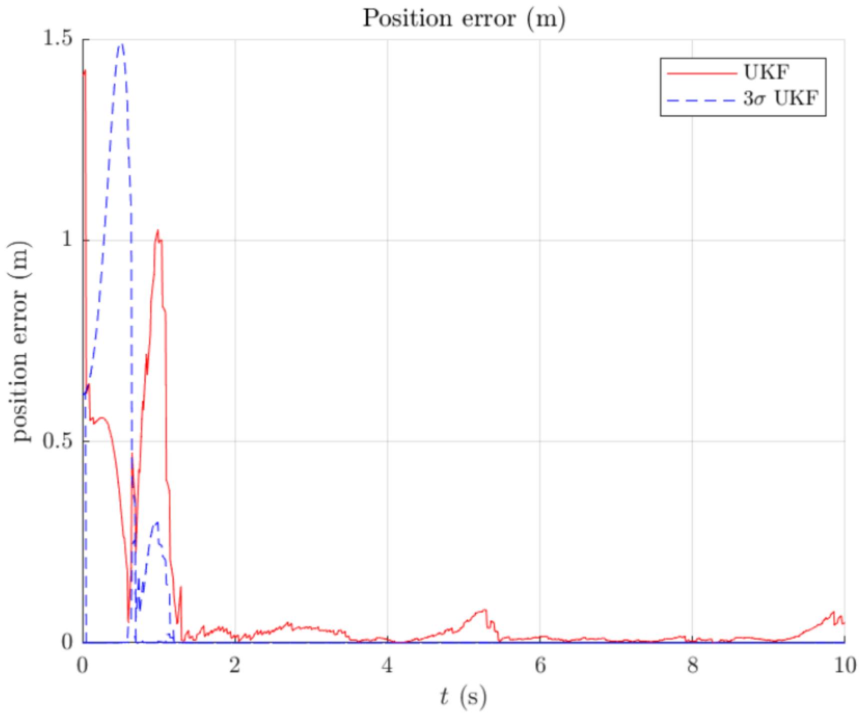


Figure 4: Total Error

This endeavor involves adapting the filter to handle the combined dynamics of rotational and translational motions. The choice of appropriate state variables, consideration of multiple noise sources, and optimization of filter parameters become crucial in achieving accurate estimations in these complex geometries.

The outcomes of implementing the Kalman Filter in tuned mass systems and complex geometries hold promise for improving structural control mechanisms and gaining insights into the nuanced behavior of structures in real-world applications.

Further research can focus on enhancing the current methodology employed in this study. This includes refining the UKF parameters, exploring variations of the filter, and investigating

References

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[5] M. Brossard, A. Barrau, and S. Bonnabel, "A Code for Unscented Kalman Filtering on Manifolds (UKF-M)," 2019.